Darcy's Law

Lesson Plan #1: How does water move through an aquifer?

Objectives: Learn about Darcy's Law and how water moves through an aquifer.

- What is hydraulic conductivity?
- How to determine the hydraulic conductivity of aquifer material?
- How is hydraulic conductivity related to ground water velocity?
- How to calculate ground water velocity?
- How do you obtain ground water chemistry and interpret the results?

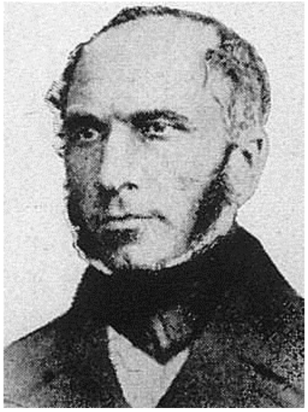
Grades: **10– 12**

Materials:

- Calculator/spreadsheet
- Darcy Tube

Darcy's Law

Henry Darcy was a French engineer living in Dijon during the mid-1800's. During this time Darcy began to conduct experiments with columns of sand to understand how water moves through the material. The results of his experiment are formulated in Darcy's Law.



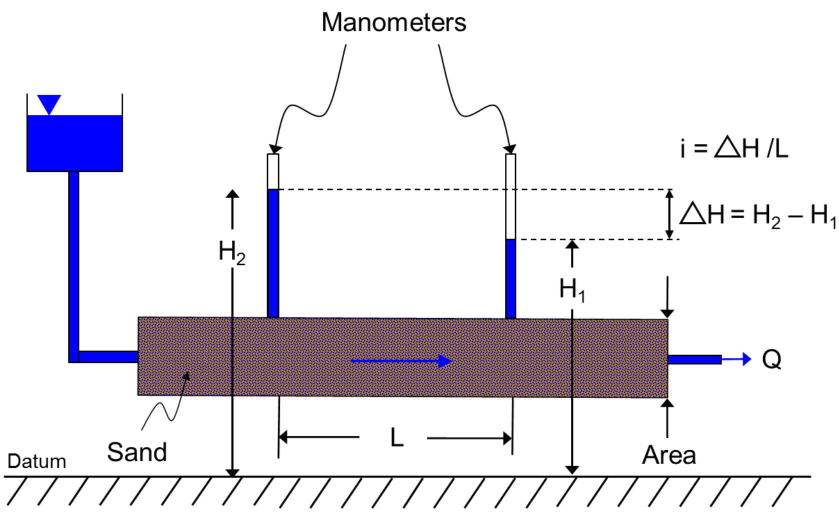
Darcy's Law is one of the fundamental equations used in hydrogeology to describe the movement of water through an aquifer and shows that discharge is proportional to the hydraulic gradient and the aquifer material.

Darcy's Law is as follows;

$$Q = kiA$$

Where;

Q = Discharge in units of length cubed per unit time (L^3/T)
K = Hydraulic conductivity in units of length per unit time (L/T)
i = Hydraulic gradient in units of length per length (L/L)
A = Cross sectional area in units of length squared (L^2)



The experiment that Darcy performed is similar to what is shown in the adjacent figure. A column is filled with sand. Manometers, which are hollow tubes used to measure water height (pressure) , are inserted into points along the column. Water is introduced into the upper end of the column and after some time water will begin to flow out the lower end. Water will push up into the manometers to heights that will indicate the gradient. What Darcy found was that he had to multiply the iA term on the right hand side by a proportionality constant in order to equal Q. The proportionality constant is termed the hydraulic conductivity and is dependent on the type of material placed into the column. Fine sand will have a lower hydraulic conductivity then coarse gravel.

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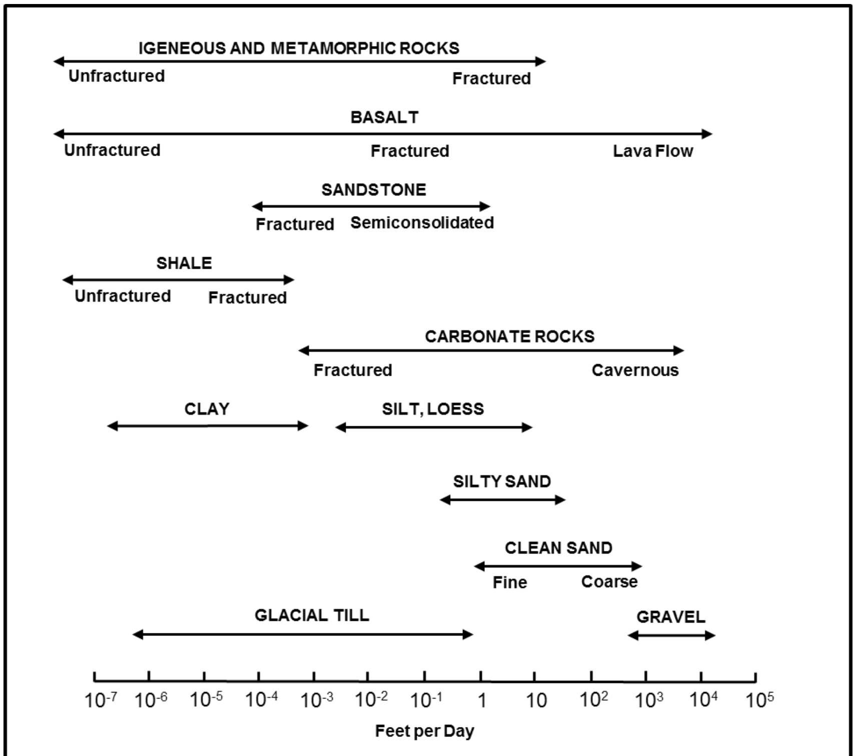


From Heath, 1983. Basic ground-water hydrology, US Geological Survey Water Supply Paper 2220

The adjacent figure describes the range of hydraulic conductivity of various aquifer materials.

We can use Darcy's Law to calculate the velocity of ground water. The water discharged through the end of a pipe is described as;

$$Q = vA$$

where

$Q =$ Discharge (L^3/T)
$v =$ water velocity (L/T)
$A =$ Cross sectional area (L^2)

If we rearrange the equation to solve for velocity we have;

$$v = Q/A$$

If we rearrange Darcy's law to solve for Q/A we get

$$Q/A = ki$$

Substituting this into the velocity equation above we have; the velocity of ground water moving through an aquifer;

$$v = ki$$

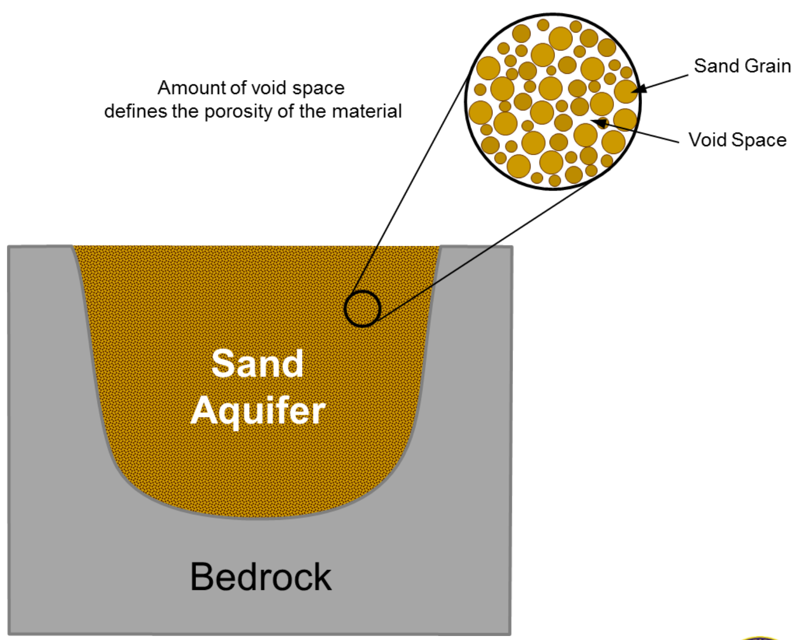
But the area of the pipe is completely open where as the area of an aquifer is not since the aquifer is composed of sand and gravel or rock. To correct for this we need to account for the much smaller open area between all the sand or gravel or rock fractures that allow water to move through.

The pore spaces of the aquifer that water travels through is termed the effective porosity (ne). The effective porosity is the open area of the aquifer divided by the area taken up by the sand and gravel. Typical range of effective porosity values for sand or gravel is 0.25 –0.35.

ne = Area of aquifer / Area of sand and gravel

To account for the smaller open area we divide the velocity equation above by the effective porosity to calculate the correct ground water velocity;

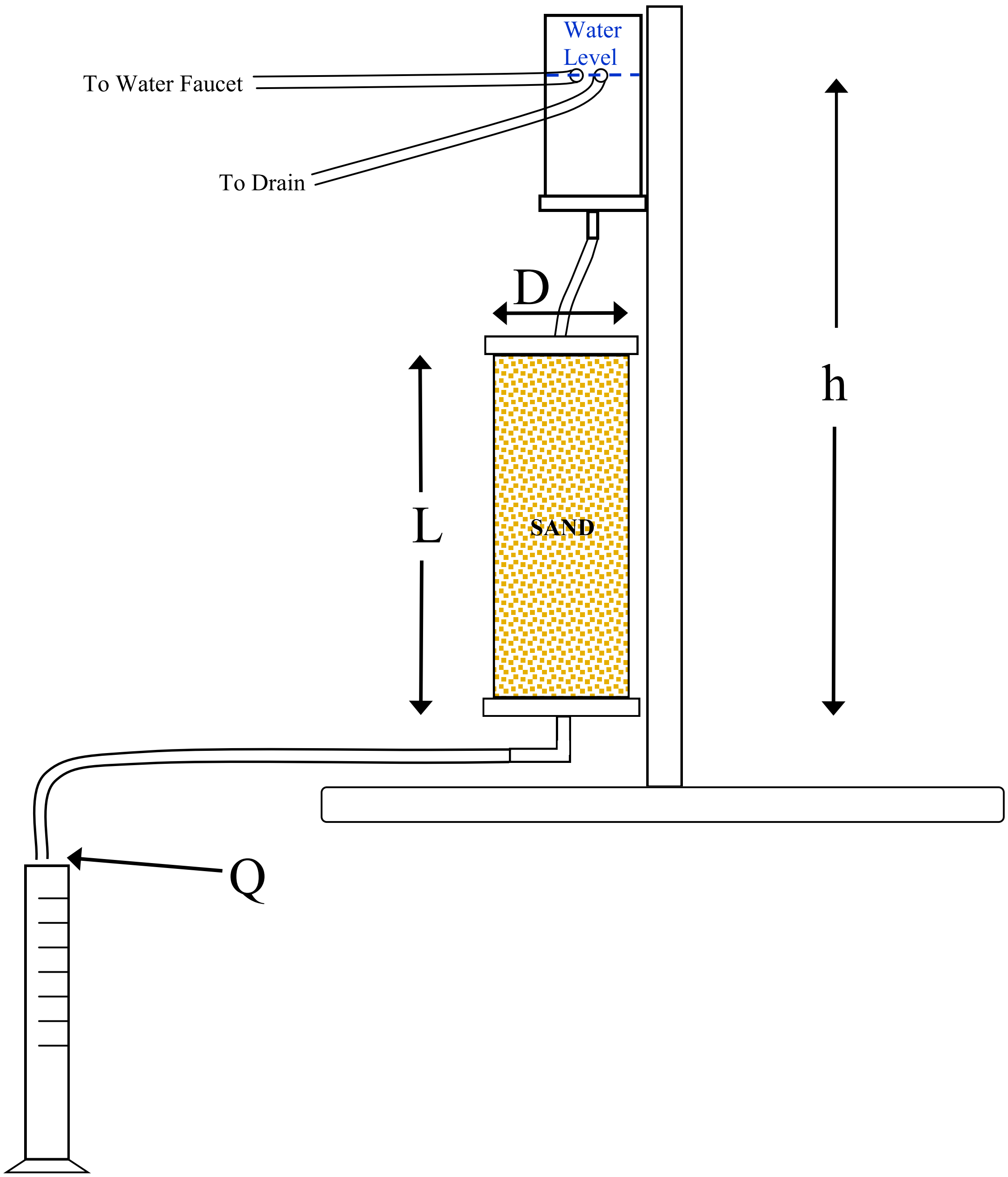
$$v = Ki/n_e$$



Activities:

Determine the hydraulic conductivity of the sample using the Darcy tube (permeameter).

Assemble the permeameter as follows:



Darcy's Law

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Activities:

1. Turn water on to funnel until equal return to drain and water level is holding steady

2. Cycle water through the permeameter until all the air has been displaced and you are receiving a steady flow.

3. Measure the lengths of h, L and D (see #2) and enter values on the attached table

4. Measure the volume of water (Q) that flows from the permeameter over a measured length of time (t) and enter the values on the attached table. Complete this a number of times.

5. Convert all the length measurements to feet and time to days.

6. Calculate the hydraulic conductivity (ft/day) for each Q and t measurement using the rearranged Darcy's Law equation;

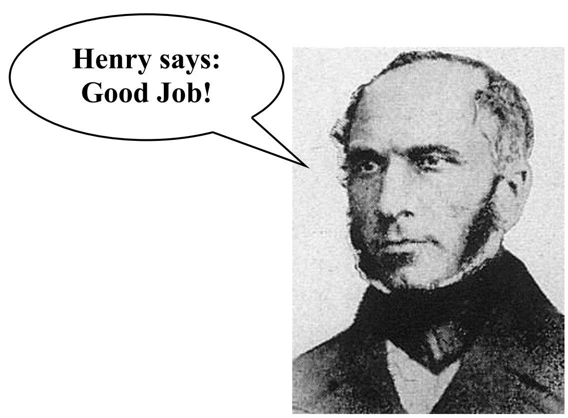
   $K = QL/hAt$

7. Calculate the average the hydraulic conductivity value.

8. Compare to the chart on page 2 of the lesson.

9. Using the K value from above and an effective porosity value from page 2, calculate the velocity of the water moving through the Darcy tube.

   $V = Ki/ne$
   Where; i = Gradient (ft/ft) = h (ft) / L (ft)
   ne = effective porosity ≈ 0.25



Test Data							Calculations					
Test No.	Material	Head Difference (h) (in)	Sample Length (L) (in)	Cylinder Diameter (D) (in)	Discharge Time* (t) (seconds)	Water Volume (V) (ml)	Head Difference (h) (ft)	Sample Length (L) (ft)	Cylinder Area (A) (ft^2)	Discharge Time* (t) (days)	Discharge Volume (Q) (ft^3)	Hydraulic Conductivity (K) (ft/day)
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												

* The time necessary to discharge the water volume (V) from permeameter

Head Difference (ft) = Head Difference (in) / 12 in/ft

Sample Length (ft) = Head Difference (in) / 12 in/ft

Radius (ft) = Diameter (in) /2/ 12in/ft

Cylinder Area (ft^2) = π * (Radius (ft))^2 π = 22/7 ≈ 3.1429

Discharge Time (days) = Seconds /86,400 seconds /day

Water Volume (ft^3) = Mililiters / 28,317 ml/ft^3

Hydaulic Conductivity (ft/day) = QL/hAt = [Discharge Volume (ft^3) * Length (ft)] / [(Head Difference (ft) * Area (ft^2) * Discharge Time (days)]